Homework 3

CS 3330 : Algorithms (Summer 2018)
Due in class on Thu, July 5

Instructions:

• Collaboration is permitted, but you must write the solutions by yourself without assistance, and be ready to explain them orally to a member of the course staff if asked. You must also identify your collaborators. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.

• For problems that require you to provide an algorithm, you must give a precise description of the algorithm, together with a proof of correctness and an analysis of its running time. You may use algorithms from class as subroutines. You may also use any facts that we proved in class.

• Check the late policy from course Web page.

1. (Oral history) Chapter 3, problem 12.
   Hint: Construct a directed graph $G$, such that the test of consistency turns out to be equivalent to testing whether $G$ is acyclic. Note that, if $G$ does not have a cycle, then it has a topological ordering. Explain how the existence of an ordering implies consistency in the given pieces of information.

2. (SSSP problem) Check the following AlgoWiki entry on the Single Source Shortest Path (SSSP) problem.
   [https://algowiki-project.org/en/Single_Source_Shortest_Path_(SSSP)]
   Let $G$ be a directed, edge-weighted graph such that every edge has a weight that belongs to the set $0, 1, \cdots, W$, where $W$ is a non-negative integer.
   (a) Carefully describe a modified implementation of Dijkstra’s algorithm so that the SSSP problem can be solved in $O(n \cdot W + m)$ time for this type of graphs. Here, as usual, the input graph has $n$ vertices and $m$ edges.
   (b) Separately argue that the worst case running time of your algorithm is indeed $O(n \cdot W + m)$. 
3. (Greedy trucking) Chapter 4, problem 3.

4. (Triathlon exercise) Chapter 4, problem 6.

5. (Video streaming) Chapter 4, problem 12.

6. (Smallest complete supervisory committee) Chapter 4, problem 15.